Innovations

Applications of Generalized Pare to Distribution for Deteriorating EPQ Model with Selling Price Dependent Demand

K. Srinivasaro[1], B. Punyavathi[1]

[1]Department of Statistics, Andhra University, Visakhapatnam-530003

Abstract

Problem: *This paper address the utility of Generalized Pareto Distribution in scheduling the production quantity and selling price of an inventory system. The stochastic inventory systems are useful for developing EPQ models in which the replenishment and life of the commodity are random.* **Methodology:** *Here it is assumed that the replenishment (production) as well as life time of the commodity follow Generalized Pareto Distributions with different parameters. The Generalized Pareto Distribution includes different types of production rates and deterioration rates.* **Findings:** *It is also assumed that the demand is a function of selling price. With suitable cost considerations and assuming shortages are allowed and fully backlogged the instantaneous state of inventory is derived.* **Conclusion:** *By maximizing the total profit rate function the optimal ordering quantity, selling price and production up and down times are derived. The sensitivity analysis of the model reveals that the random production distribution and deterioration distribution parameters have significant influence on the optimal operating policies of the model. This model also includes some of the earlier Economic Production quantity Models as particular cases. This model also includes the models without shortages as a limiting case.*

Keywords: *EPQ models, Selling Price Dependent Demand, Random Decay, Random Production.*

Introduction

Recently the stochastic modeling of Economic Production Quantity gained due importance, due to its utility in developing optimal production schedules . In derives the Economic Production Quantity models it is customary to consider that the various different assumptions on the constituent components of the model namely production (replenishment), life time of the commodity and demand pattern Srinivasa rao et al. [16], several authors developed Economic Production Quantity models for deteriorating items with random life time. The deterioration may be due to random factors such as environment conditions, storage facilities, processing methodology, quality of raw material. A good deal of literature on inventory models for deteriorating items has been presented by Nahmias [9], Raafat [12], Goyal, S. K and Giri [7], Ruxian, LI., Lan, H. and Mawhinney [13], Pentico [11], to develop the stochastic Economic Production Quantity models for deteriorated items the researcher consider differently production distribution in characterizing, the life time of the commodity Ghare [4], Shah and jaiswal [14], life time of the commodity, Cohen [2], Aggarwal [1], Pal [10], Giri, B. C and Chaudhuri [5], and others have develops Economic Production Quantity models with the assumptions, that the life time of the commodity is exponential.

Tadikamalla [19] assumed that the life time of the commodity follows Gamma Distribution. Covert, R.P. and Philip [3], Goel, V. P and Aggarwal [6], Venkata Subbaiah et al. [20], have consider the life time of the commodity follows a Wibull Distribution . Srinivasa Rao et al. [17], developed inventory models with Generalized Pareto life time. The Generalized Pareto life time

distribution is a more useful distribution which characterizes the increasing the rate of deterioration more efficiently. In all these papers it is assumed that the production rate is finite constant. But in many practical situation it is observed that the production process is a random due to influences of various random factors such as skill level of the manpower, quality of raw material, power supply, breakdowns, maintenance.
Srinivasa Rao, K, Punyavathi,B [22], developed Inventory model with generalized pareto rate of replenishment having time dependent demand.

Recently Sridevi et al [15], Srinivasa rao et al [18], Lakshmana Rao et al [19] have Economic production quantity models with random production. They assumed the production process is characterized by Wibull Distribution. Srinivasa Rao, K, Punyavathi,B [21] assumed On an EPQ model with generalized pareto rate of replenishment and deterioration and constant demand.

Very little work has been reported regarding stochastic Economic Production Quantity models with Generalized Pareto Rate of Production and deterioration. The Generalized Pareto Rate of Production is a common phenomenon in production processes where the production rate is time dependent. Hence in this application we develop and analyze a stochastic Economic production quantity models with Generalized Pareto Rate of Production and deterioration and production having Selling Price Dependent Demand. The Selling Price Dependent Demand is a common phenomenon server of the production scheduling problems in which demand is a function of selling price. Using stochastically differential equations the instantaneous inventory is derived and analyzed. The rest of the paper is organized as follows.

Assumptions

The following assumptions are made for developing the model.

i) The demand rate is a selling price dependent demand and is of the form

$$\lambda(s) = (\eta - \theta s); \quad 0 < \theta < 1 \tag{1}$$

Where 'η' and 'θ' are parameters and 's' is a selling price dependent demand.

ii) The replenishment is finite and follows a two parameter Weibull distribution with probability density function

$$f(t) = \frac{1}{\alpha}\left(1 - \frac{\beta t}{\alpha}\right)^{\frac{1}{\beta}-1}; (v \neq 0);\ 0 < t < \frac{\alpha}{\beta}.$$

Therefore, the instantaneous rate of replenishment is

$$k(t) = \frac{f(t)}{1-F(t)} = \frac{1}{\alpha - \beta t};\ t > 0 \tag{2}$$

iii) Lead time is zero

iv) Cycle length, T is known and fixed

v) Shortages are allowed and fully backlogged

vi) A deteriorated unit is lost

vii) The deterioration of the item is random and follows a generalized Pareto distribution.

Then the instantaneous rate of deterioration is

$$h(t) = \frac{1}{\lambda_1 - \lambda_2 t};\ 0 < t < \frac{\lambda_1}{\lambda_2} \tag{3}$$

Notations

Q: Ordering quantity in one cycle

A: Ordering cost

C: Cost per unit

h: Inventory holding cost per unit per unit time

π: Shortages cost per unit time

s: Selling price per unit

θ,: Parameters of Weibull distribution

P(t): Profit rate function per unit time.

I(t): Inventory level at any time 'T'

Inventory model with shortages

Consider an inventory system in which the stock level is zero at time t=0. The Stock level increases during the period $(0, t_1)$, due to excess production after fulfilling the demand and deterioration. The production stops at time t_1 when the stock level reaches S. The inventory decreases gradually due to demand and deterioration in the interval (t_1, t_2). At time t_2 the inventory reaches zero and back orders accumulate during the period (t_2, t_3). At time t_3 the production again starts and fulfils the backlog after satisfying the demand. During (t_3, T) the back orders are fulfilled and inventory reaches zero at the end of the cycle T. The Schematic diagram representing the instantaneous state of inventory is given in Figure1

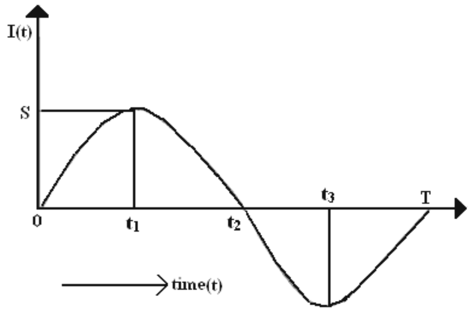

Fig 1: Schematic diagram representing the inventory level with shortages.

The differential equations governing the in the cycle time [0, T] are:

$$\frac{d}{dt}I(t) + \left(\frac{1}{\lambda_1-\lambda_2 t}\right)I(t) = \frac{1}{\alpha-\beta t} - (\eta - \theta s) \quad ; \quad 0 \le t \le t_1 \tag{4}$$

$$\frac{d}{dt}I(t)\frac{I(t)}{\lambda_1-\lambda_2 t} = -\eta - \theta s \quad ; \quad t_1 \le t \le t_2 \tag{5}$$

$$\frac{d}{dt}I(t) = -\eta - \theta s \quad ; \quad t_2 \le t \le t_3 \tag{6}$$

$$\frac{d}{dt}I(t) = \frac{1}{\alpha-\beta t} - \eta - \theta s \quad ; \quad t_3 \le t \le T \tag{7}$$

Solving the differential equations (4) and (7) using the initial conditions,

I (0) = 0, I (t_1) = S, I (t_2) = 0 and I (T) = 0, one can obtain the on hand inventory at time 't' as

$$I(t) = \left(\frac{\lambda_1-\lambda_2 t}{\lambda_1-\lambda_2 t_1}\right)^{\frac{1}{\lambda_2}}\left[S - \frac{\eta-\theta s}{\lambda_2-1}(\lambda_1-\lambda_2 t_1)\right] + \frac{\eta-\theta s}{\lambda_2-1}(\lambda_1-\lambda_2 t) - (\lambda_1-\lambda_2 t)^{\frac{1}{\lambda_2}}\int_t^{t_1}\frac{(\lambda_1-\lambda_2 t)^{-\frac{1}{\lambda_2}}}{\alpha-\beta u}du;\ 0 \le t \le t_1 \tag{8}$$

$$I(t) = \left(\frac{\lambda_1-\lambda_2 t}{\lambda_1-\lambda_2 t_1}\right)^{\frac{1}{\lambda_2}}\left[S - \frac{\eta-\theta s}{\lambda_2-1}(\lambda_1-\lambda_2 t_1)\right] + \frac{\eta-\theta s}{\lambda_2-1}(\lambda_1-\lambda_2 t)\,;\ 0 \le t \le t_1 \tag{9}$$

$$I(t) = (\eta-\theta s)(t_2 - t)\,;\quad t_2 \le t \le t_3 \tag{10}$$

$$I(t) = \frac{1}{\beta}\ln\left(\frac{\alpha-\beta T}{\alpha-\beta t}\right) + (\eta-\theta s)\,(T-t)\ ;\quad t_3 \le t \le T \tag{11}$$

Stock loss due to deterioration in the interval (0, t) is

$$L(t) = \int_0^t k(t)dt - \int_0^t (\eta-\theta s)dt - I(t)\,;\qquad 0 \le t \le t_2$$

This implies

$$L(T) = \ln\left(1-\frac{\beta}{\alpha}t\right)^{-\frac{1}{\beta}} - (\eta-\theta s)t - \left\{\left(\frac{\lambda_1-\lambda_2 t}{\lambda_1-\lambda_2 t_1}\right)\left[s-\frac{\eta-\theta s}{\lambda_2-1}(\lambda_1-\lambda_2 t_1)\right]\right.$$
$$\left.+\frac{\eta-\theta s}{\lambda_2-1}(\lambda_1-\lambda_2 t)-(\lambda_1-\lambda_2 t)^{\frac{1}{\lambda_2}}\int_t^{t_1}\frac{(\lambda_1-\lambda_2 u)^{-\frac{1}{\lambda_2}}}{\alpha-\beta u}du\right\}; \quad 0\le t\le t_1$$
$$\ln\left(1-\frac{\beta}{\alpha}t_1\right)^{-\frac{1}{\beta}} - (\eta-\theta s)t - \left\{\left(\frac{\lambda_1-\lambda_2 t}{\lambda_1-\lambda_2 t_1}\right)^{\frac{1}{\lambda_2}}\left[s-\frac{\eta-\theta s}{\lambda_2-1}(\lambda_1-\lambda_2 t_1)\right]\right.$$
$$\left.+\frac{\eta-\theta s}{\lambda_2-1}(\lambda_1-\lambda_2 t)\right\}; \quad t_1\le t\le t_2$$

Stock loss due to deterioration in the cycle of length T is

$$L(T) = \ln\left(1-\frac{\beta}{\alpha}t_1\right)^{-\frac{1}{\beta}} - (\eta-\theta s)t_2$$

Ordering quantity Q in the cycle of length T is

$$Q = \int_0^{t_1}k(t)dt + \int_{t_3}^{T}k(t) = \frac{1}{\beta}ln\left[\frac{\alpha(\alpha-\beta t_3)}{(\alpha-\beta t_1)(\alpha-\beta T)}\right] \quad (12)$$

From equation (4.3.5) and using the condition I (0) = 0, we obtain the value of 'S' as

$$S = (\lambda_1-\lambda_2 t_1)^{\frac{1}{\lambda_2}}\int_0^{t_1}\frac{1}{\alpha-\beta u}(\lambda_1-\lambda_2 u)^{-\frac{1}{\lambda_2}}du + \frac{(\eta-\theta s)}{(\lambda_2-1)}\left[(\lambda_1-\lambda_2 t_1)-\lambda_1^{-\frac{1}{\lambda_2}+1}(\lambda_1-\lambda_2 t_1)^{\frac{1}{\lambda_2}}\right] \quad (13)$$

From equation (9) and using the condition I (t_2) = 0, one can get

$$t_2 = \frac{1}{\lambda_2} - \left(\frac{(\eta-\theta s)(\lambda_1-\lambda_2 t_1)^{\frac{1}{\lambda 2}}}{(\eta-\theta s)(\lambda_1-\lambda_2 t_1)-S(\lambda_1-1)}\right)^{\frac{\lambda 2}{1-\lambda 2}} \quad (14)$$

Substituting the values of 'S' given in equations (13) in (14), one can get t_2 in terms of t_1 as

$$t_2 = \frac{1}{\lambda_2}\left[\lambda_1 - \frac{(\eta-\theta s)\frac{\lambda_2}{1-\lambda_2}}{(\eta-\theta s)(\lambda_1-\lambda_2 t_1)-S(\lambda_2-1)}\right] = x(t_1,s) \quad (15)$$

When t = t_3, then equations (10) and (11) become

$$I(t_3) = (\eta-\theta s)(t_2-t_3) \quad (16)$$

And

$$I(t_3) = \frac{1}{\beta}\ln\left(\frac{\alpha-\beta T}{\alpha-\beta t_3}\right) + (\eta-\theta s)(T-t_3) \quad (17)$$

Equating the equations and on simplification, one can get

$$t_3 = \frac{1}{\beta}\left[\alpha-(\alpha-\beta T)e^{\beta(\eta-\theta s)(T-t_2)}\right] \quad (18)$$

Substituting the values of t_2 from (15) in equation (16), one can get t_3 in terms of t_1

$$t_3 = \left(T^{\beta} - \frac{(\eta-\theta s)}{\alpha}\left(T-x(t_1,s)\right)\right)^{\frac{1}{\beta}}$$

Let $y(t_1,s) = \frac{(\eta-\theta s)}{\alpha}\left(T-x(t_1,s)\right)$ (19)

$$t_3 = \left(T^{\beta} - y(t_1,s)\right)^{\frac{1}{\beta}} \quad (20)$$

Let $K(t_1,t_2,t_3,s)$ be the total cost per unit time. Since the total cost is the sum of the set up cost, cost of the units, the inventory holding cost and shortage cost, the total cost per unit time becomes

$$K(t_1,t_2,t_3,s) = \frac{A}{T} + \frac{CQ}{T} + \frac{h}{T}\left(\int_0^{t_1}I(t)dt + \int_{t_1}^{t_2}I(t)dt\right) + \frac{\pi}{T}\left(\int_{t_2}^{t_3}-I(t)dt + \int_{t_3}^{T}-I(t)dt\right) \quad (21)$$

Substituting the values of I (t) and Q in equation (19) and on simplification one can obtain K (t_1, t_2, t_3, s) as

$$K(t_1,t_2,t_3,s) = \frac{A}{T} + \frac{C}{T}\frac{1}{\beta}\ln\left[\frac{\alpha(\alpha-\beta t_3)}{(\alpha-\beta t_1)(\alpha-\beta T)}\right]$$

$$+\frac{h}{T}\left\{-\int_0^{t_1}(\lambda_1-\lambda_2 t)^{\frac{1}{\lambda_2}}\left(\int_t^{t_1}\frac{1}{\alpha-\beta u}(\lambda_1-\lambda_2 u)^{-\frac{1}{\lambda_2}}du\right)dt\right.$$

$$+\frac{S}{\lambda_2+1}\left[\frac{\lambda_1^{\frac{1}{\lambda_2}+1}}{(\lambda_1-\lambda_2 t_1)^{\frac{1}{\lambda_2}}}-\frac{(\lambda_1-\lambda_2 t_2)^{\frac{1}{\lambda_2}}}{(\lambda_1-\lambda_2 t_1)^{\frac{1}{\lambda_2}}}(\lambda_1-\lambda_2 t_2)\right]$$

$$+\frac{\eta-\theta s}{(\lambda_2-1)(\lambda_2+1)}\left[(\lambda_1-\lambda_2 t_1)^{-\frac{1}{\lambda_2}+1}(\lambda_1-\lambda_2 t_2)^{\frac{1}{\lambda_2}+1}-\frac{\lambda_1^{\frac{1}{\lambda_2}+1}(\lambda_1-\lambda_2 t_1)^{1-\frac{1}{\lambda_2}}}{(\lambda_1-\lambda_2 t_1)^{\frac{1}{\lambda_2}}}\right]$$

$$\left.+\frac{\eta-\theta s}{(\lambda_2-1)}\left[\lambda_1 t_2-\frac{\lambda_2 t_2^2}{2}\right]\right\}$$

$$+\frac{\pi}{T}\left\{\frac{\eta-\theta s}{2}\left[(t_2^2-T^2)+2t_3(T-t_2)\right]-\frac{T-t_3}{\beta}-\frac{\alpha-\beta t_3}{\beta^2}\ln\left(\frac{\alpha-\beta T}{\alpha-\beta t_3}\right)\right\} \qquad (22)$$

Let $P(t_1,t_3,s)$ be the profit rate function. Since the profit rate function is the total revenue per unit minus total cost per unit time, we have

$$P(t_1,t_3,s)=s(\eta-\theta s)-K(t_1,t_2,t_3,s) \qquad (23)$$

Substituting the equations (13), (15), (18) and (20) in equation (21), one can get the profit rate function in terms of 't_1' and 's' as

$$P\,(t_1,s)=s(\eta-\theta s)-\frac{A}{T}-\frac{C}{T}\left[\frac{1}{\beta}\ln\left(\frac{\alpha}{\alpha-\beta t_1}\right)+\frac{y(t_1,s)}{\beta}\right]$$

$$-\frac{h}{T}\left\{-\int_0^{t_1}(\lambda_1-\lambda_2 t)^{\frac{1}{\lambda_2}}\left(\int_t^{t_1}\frac{1}{\alpha-\beta u}(\lambda_1-\lambda_2 u)^{-\frac{1}{\lambda_2}}du\right)dt\right.$$

$$+\left[\frac{1}{\lambda_2+1}\left[\lambda_1^{\frac{1}{\lambda_2}+1}\left[\lambda_1-\lambda_2 x(t_1,s)^{\frac{1}{\lambda_2}+1}\right]\int_0^{t_1}\frac{1}{\alpha-\beta u}(\lambda_1-\lambda_2 u)^{-\frac{1}{\lambda_2}}du+\frac{\eta-\theta s}{(\lambda_2-1)(\lambda_2+1)}\right.\right.$$

$$\left.\left.\left[\lambda_1^{1-\frac{1}{\lambda_2}+1}\left[\lambda_1-\lambda_2 x(t_1,s)^{\frac{1}{\lambda_2}+1}-{\lambda_1}^2\right]+\frac{\eta-\theta s}{\lambda_2-1}\left[\lambda_1 x(t_1,s)-\frac{\lambda_2(x(t_1,s)^2)}{2}\right]\right]\right\}$$

$$-\frac{\pi}{T}\left\{\frac{\eta-\theta s}{2}\left[x(t_1,s)^2-T^2\right]+2\left[\frac{\alpha}{\beta}-\frac{\alpha-\beta T}{\beta}e^{y(t_1,s)}\right]\left[T-x(t_1,s)\right]\right.$$

$$\left.-\frac{T}{\beta}+\frac{\alpha}{\beta^2}-\frac{\alpha-\beta T}{\beta^2}e^{y(t_1,s)}\left[1-y(t_1,s)\right]\right\} \qquad (24)$$

Optimal pricing and ordering policies of the model with shortages

In this section we obtain the optimal policies of the production model under study. To find the optimal values of t_1, s and s we obtain the first order partial derivatives of $P(t_1,t_3,s)$ given in equation (17) with respect to t_1, s and s and equate them to zero. The condition for maximization of $P(t_1,s)$ is

$$D=\begin{bmatrix}\dfrac{\partial^2 P(t_1,s)}{\partial t_1^2} & \dfrac{\partial^2 P(t_1,s)}{\partial t_1\partial s}\\ \dfrac{\partial^2 P(t_1,s)}{\partial t_1\partial s} & \dfrac{\partial^2 P(t_1,s)}{\partial s^2}\end{bmatrix}<0$$

Using equation (15), one can get

$$\frac{\partial t_2}{\partial t_1}=\frac{\partial x(t_1,s)}{\partial t_1}$$

Let Z_1 (t_1, s) = $\frac{\partial x(t_1,s)}{\partial t_1}$ (25)

From equations (15) and (23), one can get

$$Z_1(t_1, s) = (\eta - \theta s)^{\frac{\lambda 2}{1-\lambda 2}} \frac{1}{\alpha - \beta u}(\lambda_1 - \lambda_2 u)^{-\frac{1}{\lambda_2}} \left[\left((\lambda_2 - 1) \int_0^{t_1} \frac{1}{\alpha - \beta u}(\lambda_1 - \lambda_2 u)^{-\frac{1}{\lambda_2}} du - (\eta - \theta s)\lambda_1^{-\frac{1}{\lambda_2}+1} \right)^{\frac{1}{(\lambda 2 - 1)}} \right] \quad (26)$$

Let $Z_2(t_1, s) = \frac{\partial x(t_1,s)}{\partial t_1}$ (27)

Using equation (15), equation (25) can be written as

$$Z_2(t_2, s) = \frac{\theta}{\lambda_2 - 1} \left| \lambda_1^{-\frac{1}{\lambda_2}+1} \left((\eta - \theta s) \right)^{\frac{1}{(\lambda 2-1)}} \left((\eta - \theta s)\lambda_1^{-\frac{1}{\lambda_2}+1} - (\lambda_2 - 1) \int_0^{t_1} \frac{1}{\alpha - \beta u}(\lambda_1 - \lambda_2 u)^{-\frac{1}{\lambda_2}} du \right) - (\eta - \theta s)^{\frac{1-2\lambda 2}{\lambda 2 - 1}} \left((\eta - \theta s)\lambda_1^{-\frac{1}{\lambda_2}+1} - (\lambda_2 - 1) \int_0^{t_1} \frac{1}{\alpha - \beta u}(\lambda_1 - \lambda_2 u)^{-\frac{1}{\lambda_2}} du \right)^{\frac{\lambda 2}{1-\lambda 2}} \right] \quad (28)$$

$Z_3(t_1, s) = \frac{\partial x(t_1,s)}{\partial s}$ (29) where, $x(t_1, s)$ is given in equation (17)

Using equations (17) and (27), $Z_3(t_1, s)$ can be written as

$$Z_3(t_1, s) = e^{\beta(\eta - \theta s)}(T - x(t_1, s))\left[-\beta(\eta - \theta s)(Z_2(t_1, s))\right] - \left[\beta\theta(T - x(t_1, s))\right] \quad (30)$$

Differentiate $P(t_1, s)$ given in equation (22) with respect to t_1, using equations (17) and (24) and equating to zero, one can get

$$C\left(\frac{1}{\alpha - \beta t_1} + \frac{1}{\beta}\frac{\partial}{\partial t_1}y(t_1, s)\right) \Bigg\{ \lambda_1 - \lambda_2 x(t_1, s)^{\frac{1}{\lambda_2}} \left(Z_1(t_1, s)\right) \int_0^{t_1} \left[\frac{1}{\alpha - \beta u}(\lambda_1 - \lambda_2 u)^{-\frac{1}{\lambda_2}} du\right] + \frac{1}{\lambda_2 + 1}\left| \lambda_1^{\frac{1}{\lambda_2}+1}\left[\lambda_1 - \lambda_2 x(t_1, s)^{\frac{1}{\lambda_2}+1}\right]\right| + \frac{\eta - \theta s}{\lambda_2 - 1}\left(Z_1(t_1, s)\right)\left[\lambda_1 - \lambda_1^{1-\frac{1}{\lambda_2}} - x(t_1, s)\right]\Bigg\}$$

$$\left\{(\eta - \theta s)Z_1(t_1, s)\left[x(t_1, s) - \left[\frac{\alpha}{\beta} - \frac{\alpha - \beta T}{\beta}e^{y(t_1,s)}\frac{\partial}{\partial t_1}y(t_1, s)\left[T - x(t_1, s)\right]\right.\right.\right. + \frac{\alpha - \beta T}{\beta^2}y(t_1, s)e^{y(t_1,s)}\frac{\partial}{\partial t_1}y(t_1, s)\Big\} \quad (31)$$

Numerical illustration of the model with shortages:

In this section we discuss the solution procedure of the model through a numerical illustration by obtaining the production uptime, production down time, optimal selling price, optimal quantity and total profit of an inventory system. Here, it is assumed that the commodity is of deteriorating nature and shortages are allowed and fully back logged. The following parameter values one considered for illustrations:

A = 150, 200, 250, 300; α = 10, 11, 12, 13

C = 10, 11, 12, 13; β = 0.5, 0.55, 0.6, 0.65

h = 2, 2.2, 2.4, 2.6; λ_1 = 100, 102, 104, 106

π = 0.5, 0.6, 0.65; λ_2 = 10, 11, 12, 13

η = 15, 15.5, 16, 16.5; θ = 0.5, 0.51, 0.52, 0.53; T = 12.

The values of above parameters are varied further to observe the trend in optimal policies and the results obtained are shown in Table 1.

Table1 (a): Optimal values of production schedule, selling price and production for given values of the parameters and costs

A	C	h	π	T	α	β	λ_1	λ_2	n	⍰	θ	t_1	t_3	s	Q	K
150	10	2	0.5	12	10	0.5	100	10	2	15	0.5	5.324	6.99	53.178	0.691	17.143
200	10	2	0.5	12	10	0.5	100	10	2	15	0.5	5.308	6.99	53.021	0.69	21.252
250	10	2	0.5	12	10	0.5	100	10	2	15	0.5	5.293	6.99	52.863	0.689	25.361
300	10	2	0.5	12	10	0.5	100	10	2	15	0.5	5.277	6.99	52.705	0.688	29.47
150	11	2	0.5	12	10	0.5	100	10	2	15	0.5	5.325	6.988	53.176	0.691	17.202
150	12	2	0.5	12	10	0.5	100	10	2	15	0.5	5.326	6.986	53.174	0.692	17.26
150	13	2	0.5	12	10	0.5	100	10	2	15	0.5	5.326	6.984	53.171	0.692	17.319
150	10	2.2	0.5	12	10	0.5	100	10	2	15	0.5	5.353	6.99	53.448	0.693	17.624
150	10	2.4	0.5	12	10	0.5	100	10	2	15	0.5	5.381	6.99	53.76	0.695	18.272
150	10	2.6	0.5	12	10	0.5	100	10	2	15	0.5	5.381	6.99	53.764	0.759	18.262
150	10	2	0.55	12	10	0.5	100	10	2	15	0.5	5.324	6.993	53.18	0.691	17.115
150	10	2	0.6	12	10	0.5	100	10	2	15	0.5	5.324	6.995	53.182	0.691	17.087
150	10	2	0.65	12	10	0.5	100	10	2	15	0.5	5.324	6.998	53.184	0.691	17.06
150	10	2	0.5	12	11	0.5	100	10	2	15	0.5	5.321	6.994	53.179	0.593	17.038
150	10	2	0.5	12	12	0.5	100	10	2	15	0.5	5.318	6.997	53.181	0.52	16.958
150	10	2	0.5	12	13	0.5	100	10	2	15	0.5	5.316	6.999	53.182	0.464	16.894
150	10	2	0.5	12	10	0.55	100	10	2	15	0.5	5.325	6.987	53.178	0.743	17.206
150	10	2	0.5	12	10	0.6	100	10	2	15	0.5	5.325	6.984	53.179	0.807	17.274
150	10	2	0.5	12	10	0.65	100	10	2	15	0.5	5.326	6.981	53.18	0.892	17.355
150	10	2	0.5	12	10	0.5	102	10	2	15	0.5	5.324	6.99	53.18	0.691	17.147
150	10	2	0.5	12	10	0.5	104	10	2	15	0.5	5.325	6.99	53.182	0.691	17.15
150	10	2	0.5	12	10	0.5	106	10	2	15	0.5	5.325	6.99	53.183	0.691	17.154

Table1 (b): Optimal values of production schedule, selling price and production for given values of the parameters and costs

150	**10**	**2**	**0.5**	**12**	**10**	**0.5**	**100**	**11**	**2**	**15**	**0.5**	5.323	6.99	53.176	0.691	17.136
150	**10**	**2**	**0.5**	**12**	**10**	**0.5**	**100**	**12**	**2**	**15**	**0.5**	5.323	6.99	53.173	0.691	17.129
150	**10**	**2**	**0.5**	**12**	**10**	**0.5**	**100**	**13**	**2**	**15**	**0.5**	5.322	6.99	53.169	0.691	17.12
150	**10**	**2**	**0.5**	**12**	**10**	**0.5**	**100**	**10**	**2.4**	**15**	**0.5**	5.331	6.991	53.255	0.692	17.271
A	**C**	**h**	**π**	**T**	**α**	**β**	**λ_1**	**λ_2**	**n**	**[illegible]**	**θ**	**t_1**	**t_3**	**S**	**Q**	**K**
150	**10**	**2**	**0.5**	**12**	**10**	**0.5**	**100**	**10**	**2.8**	**15**	**0.5**	5.329	6.991	53.238	0.691	17.237
150	**10**	**2**	**0.5**	**12**	**10**	**0.5**	**100**	**10**	**3.2**	**15**	**0.5**	5.323	6.991	53.172	0.691	17.118
150	**10**	**2**	**0.5**	**12**	**10**	**0.5**	**100**	**10**	**2**	**15.5**	**0.5**	5.31	6.99	53.187	0.69	16.942
150	**10**	**2**	**0.5**	**12**	**10**	**0.5**	**100**	**10**	**2**	**16**	**0.5**	5.297	6.99	53.196	0.69	16.743
150	**10**	**2**	**0.5**	**12**	**10**	**0.5**	**100**	**10**	**2**	**16.5**	**0.5**	5.283	6.991	53.205	0.689	16.546
150	**10**	**2**	**0.5**	**12**	**10**	**0.5**	**100**	**10**	**2**	**15**	**0.51**	5.338	6.99	53.232	0.692	17.369
150	**10**	**2**	**0.5**	**12**	**10**	**0.5**	**100**	**10**	**2**	**15**	**0.52**	5.351	6.99	53.286	0.693	17.597
150	**10**	**2**	**0.5**	**12**	**10**	**0.5**	**100**	**10**	**2**	**15**	**0.53**	5.364	6.99	53.339	0.694	17.827

Sensitivity analysis of the model with shortages

Sensitivity analysis is carried to explore the effect of changes in model parameters and costs on the optimal policies, by varying each parameter (-15%, -10%, -5%, 0%, 5%, 10%, 15%) at a time for the model under study. The results are presented in Table 2. The relationship between the parameters and the optimal values of the production schedule is shown in Figure 1.

Table 2 (a): Sensitivity Analysis of the Model - With Shortages

Variation Parameters/Costs	Optimal Values	Change in parameters						
		-15%	-10%	-5%	0%	5%	10%	15%
A	t_1^*	5.331	5.329	5.326	5.324	5.322	5.319	5.317
	t_3^*	6.99	6.99	6.99	6.99	6.99	6.99	6.99
	s^*	53.249	53.225	53.202	53.178	53.155	53.131	53.107
	Q^*	0.692	0.691	0.691	0.691	0.691	0.691	0.691
	P^*	15.294	15.911	16.527	17.143	17.759	18.376	18.992
C	t_1^*	5.323	5.323	5.324	5.324	5.325	5.325	5.325
	t_3^*	6.994	6.993	6.991	6.99	6.989	6.988	6.987
	s^*	53.182	53.18	53.179	53.178	53.177	53.176	53.175
	Q^*	0.691	0.691	0.691	0.691	0.691	0.691	0.691
	P^*	17.055	17.084	17.114	17.143	17.172	17.202	17.231
h	t_1^*	5.28	5.295	5.31	5.324	5.338	5.353	5.367
	t_3^*	6.99	6.99	6.99	6.99	6.99	6.99	6.99
	s^*	52.729	52.88	53.03	53.178	53.326	53.448	53.617
	Q^*	0.689	0.689	0.69	0.691	0.692	0.693	0.694
	P^*	16.352	16.61	16.874	17.143	17.417	17.624	17.982
π	t_1^*	5.324	5.324	5.324	5.324	5.324	5.324	5.324
	t_3^*	6.986	6.988	6.989	6.99	6.992	6.993	6.994
	s^*	53.175	53.176	53.177	53.178	53.179	53.18	53.181
	Q^*	0.691	0.691	0.691	0.691	0.691	0.691	0.691
	P^*	17.185	17.171	17.157	17.143	17.129	17.115	17.101
α	t_1^*	5.331	5.328	5.326	5.324	5.322	5.321	5.319
	t_3^*	6.982	6.985	6.988	6.99	6.992	6.994	6.995
	s^*	53.177	53.178	53.178	53.178	53.179	53.179	53.18

	Q*	0.93	0.833	0.755	0.691	0.638	0.593	0.554
	P*	17.383	17.287	17.209	17.143	17.087	17.038	16.996
β	**t_1***	5.323	5.324	5.324	5.324	5.324	5.325	5.325
	t_3*	6.996	6.994	6.992	6.99	6.989	6.987	6.986
	s*	53.179	53.179	53.179	53.178	53.178	53.178	53.178
	Q*	0.63	0.649	0.669	0.691	0.716	0.743	0.773
	P*	17.052	17.083	17.113	17.143	17.174	17.206	17.239
λ_1	**t_1***	5.321	5.322	5.323	5.324	5.325	5.326	5.327
	t_3*	6.99	6.99	6.99	6.99	6.99	6.99	6.99
	s*	53.161	53.168	53.173	53.178	53.186	53.192	53.198
	Q*	0.691	0.691	0.691	0.691	0.691	0.691	0.691
	P*	17.105	17.12	17.133	17.143	17.152	17.161	17.170
λ_2	**t_1***	5.325	5.324	5.324	5.324	5.324	5.323	5.323
	t_3*	6.99	6.99	6.99	6.99	6.99	6.99	6.99
	s*	53.183	53.181	53.179	53.178	53.177	53.176	53.175
	Q*	0.691	0.691	0.691	0.691	0.691	0.691	0.691
	P*	17.512	17.149	17.146	17.143	17.14	17.136	17.133
n	**t_1***	5.309	5.315	5.32	5.324	5.327	5.329	5.33
	t_3*	6.99	6.99	6.99	6.99	6.99	6.99	6.99
	s*	53.012	53.081	53.136	53.178	53.21	53.236	53.247
	Q*	0.69	0.691	0.691	0.691	0.691	0.691	0.692
	P*	16.863	16.979	17.072	17.143	17.197	17.224	17.259

Table 2 (b): Sensitivity Analysis of the Model - With Shortages

Variation Parameters/Costs	**Optimal Values**	**Change in parameters**						
		-15%	**-10%**	**-5%**	**0%**	**5%**	**10%**	**15%**
⍰	**t_1***	5.385	5.365	5.345	5.324	5.304	5.283	5.262
	t_3*	6.99	6.99	6.99	6.99	6.99	6.991	6.991
	s*	53.139	53.152	53.165	53.178	53.191	53.205	53.219
	Q*	0.695	0.694	0.692	0.691	0.69	0.689	0.687
	P*	18.073	17.758	17.448	17.143	16.842	16.546	16.255
θ	**t_1***	5.22	5.255	5.29	5.324	5.358	5.391	5.432
	t_3*	6.991	6.991	6.99	6.99	6.99	6.99	6.98
	s*	52.767	52.905	53.042	53.178	53.312	53.444	53.58
	Q*	0.685	0.687	0.689	0.691	0.693	0.695	0.697
	P*	15.534	16.054	16.59	17.143	17.712	18.296	18.860

The deteriorating rate parameter 'λ_1' decrease, the optimal values of t_1*, t_3*, s*, Q*and P* are decreasing. As 'λ_1' increase, the optimal values of t_1*, t_3*, s*, Q* and P* are increases. The deteriorating parameter 'λ_2' decrease, the optimal values of t_1*, t_3*, s*, Q*and P* are increasing. As 'λ_2' increase, the optimal values of t_1*, t_3*, s*, Q* and P* are decreases. The demand rate parameter 'η' decreases, the optimal values of s* are decreasing, the optimal values of t_1* t_3*, Q* and P* increases. As parameter 'η' increases, the optimal values of s* are increasing, the optimal values of t_1*, t_3*, Q* and P* decreasing. As the demand rate parameter 'θ' decreases, the optimal values of t_1*, s*, t_3*, Q* and P* are decreasing. As the parameter 'θ' increases, the optimal values of t_1*, s*, t_3*, Q* and P* are increasing.

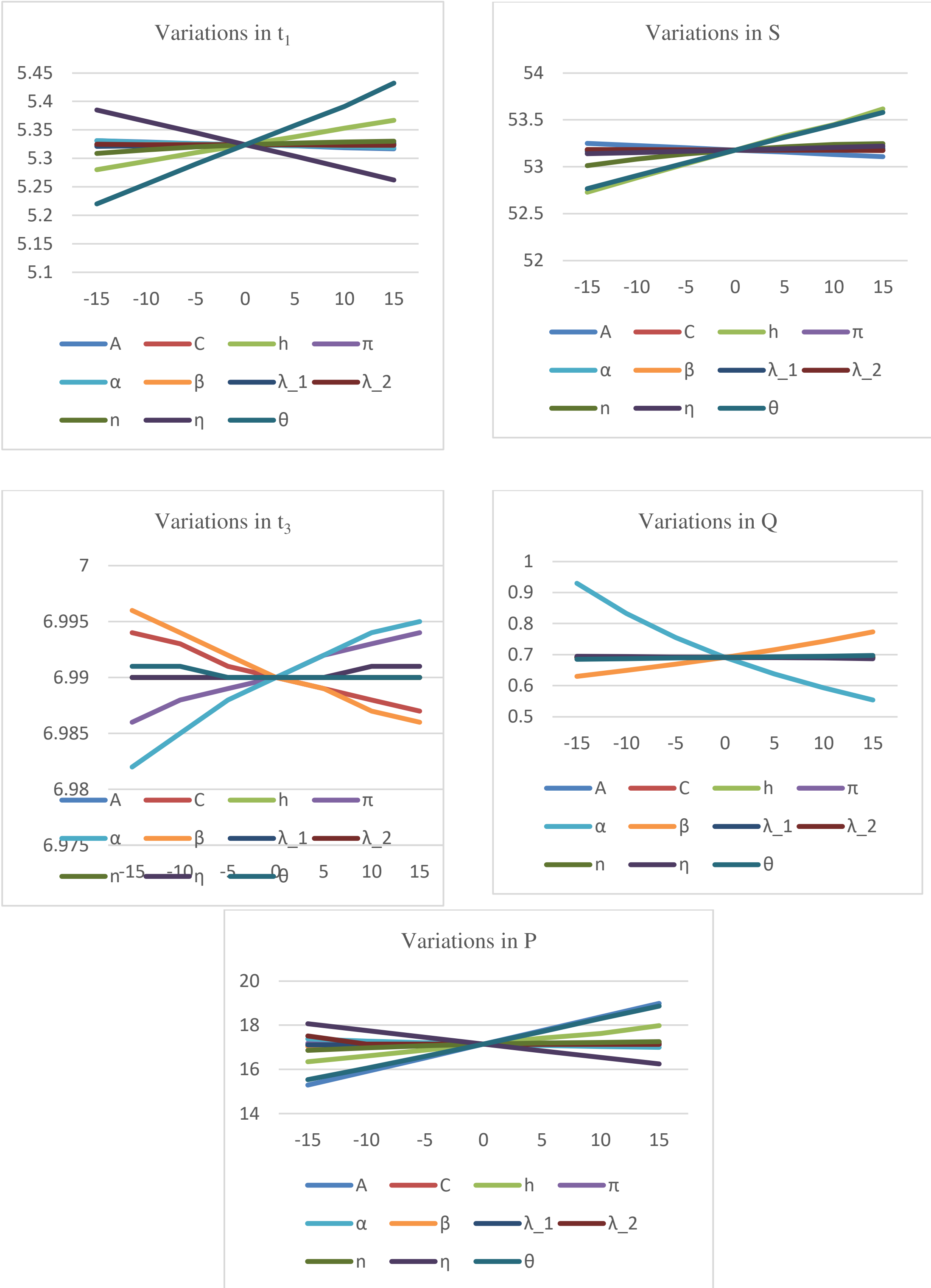

Fig 2: Relationship between parameters and optimal values without shortages

Inventory model without shortages

In this section the inventory model for deteriorating items without shortages is developed and analyzed. Here, it is assumed that shortages are not allowed and the stock level is zero at time $t = 0$. The stock level increases during the period $(0, t_1)$ due to excess production after fulfilling the demand and deterioration. The production stops at time t_1 when the stock level reaches S. The inventory decreases gradually due to demand and deterioration in the interval (t_1, T). At time T the inventory reaches zero. The Schematic diagram representing the instantaneous state of inventory is given in Figure 3.

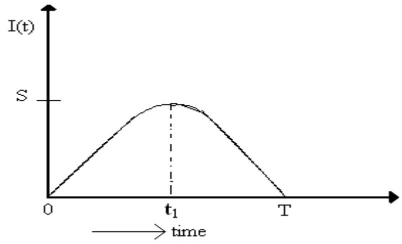

Fig 3: Schematic diagram representing the inventory level without shortages.

The differential equations governing the system in the cycle time [0, T] are:

$\frac{d}{dt}I(t)+h(t)I(t)\frac{1}{\alpha-\beta t}-(\eta-\theta s)$; $0\le t\le t_1$ (32)

$\frac{d}{dt}I(t)+h(t)I(t)=-(\eta-\theta s)$; $t_1\le t\le T$ (33)

Solving the differential equations. (32) and (33) using the initial conditions, I (0) = 0, I (t_1) = S, and I (T) = 0, one can get the on hand inventory at time't' as

$$I(t)=\left(\frac{\lambda_1-\lambda_2 t}{\lambda_1-\lambda_2 t_1}\right)^{\frac{1}{\lambda_2}}\left[S-\frac{(\eta-\theta s)}{\lambda_2-1}(\lambda_1-\lambda_2 t_1)\right]+\frac{(\eta-\theta s)}{\lambda_2-1}(\lambda_1-\lambda_2 t)$$
$$-(\lambda_1-\lambda_2 t)^{\frac{1}{\lambda_2}}\int_t^{t_1}\frac{(\lambda_1-\lambda_2 u)^{-\frac{1}{\lambda_2}}}{\alpha-\beta u}du \quad 0\le t\le t_1 \qquad (34)$$

$$I(t)=\left(\frac{\lambda_1-\lambda_2 t}{\lambda_1-\lambda_2 t_1}\right)^{\frac{1}{\lambda_2}}\left[S-\frac{(\eta-\theta s)}{\lambda_2-1}(\lambda_1-\lambda_2 t_1)\right]+\frac{(\eta-\theta s)}{\lambda_2-1}(\lambda_1-\lambda_2 t);\ t_1\le t\le T \qquad (35)$$

Stock loss due to deterioration in the interval (0, t) is

$L(t)=\int_0^t k(t)dt-\int_0^t \lambda(s,t)dt-I(t)\,, 0\le t\le T$

This implies

$$L(t)=\ln\left(1-\frac{\beta}{\alpha}t\right)^{-\frac{1}{\beta}}-(\eta-\theta s)t-\left\{\left(\frac{\lambda_1-\lambda_2 t}{\lambda_1-\lambda_2 t_1}\right)\left[s-\frac{\eta-\theta s}{\lambda_2-1}(\lambda_1-\lambda_2 t_1)\right]\right.$$
$$\left.+\frac{\eta-\theta s}{\lambda_2-1}(\lambda_1-\lambda_2 t)-(\lambda_1-\lambda_2 t)^{\frac{1}{\lambda_2}}\int_t^{t_1}\frac{(\lambda_1-\lambda_2 u)^{-\frac{1}{\lambda_2}}}{\alpha-\beta u}du\right\};0\le t\le t_1$$
$$\ln\left(1-\frac{\beta}{\alpha}t_1\right)^{-\frac{1}{\beta}}-(\eta-\theta s)t-\left\{\left(\frac{\lambda_1-\lambda_2 t}{\lambda_1-\lambda_2 t_1}\right)^{\frac{1}{\lambda_2}}\left[s-\frac{\eta-\theta s}{\lambda_2-1}(\lambda_1-\lambda_2 t_1)\right]\right.$$
$$\left.+\frac{\eta-\theta s}{\lambda_2-1}(\lambda_1-\lambda_2 t)\right\} \quad ; \qquad t_1\le t\le t_2$$

Ordering quantity Q in the cycle of length T is

$Q=\ln\left(\frac{\alpha}{\alpha-\beta t_1}\right)^{\frac{1}{\beta}}$ (36)

From equation (4.7.3) and using the condition I (0) = 0, we obtain the value of 'S' as

$S=$

$(\lambda_1-\lambda_2 t_1)^{\frac{1}{\lambda_2}}\int_0^{t_1}\left(\frac{1}{\alpha-\beta u}\right)(\lambda_1-\lambda_2 u)^{-\frac{1}{\lambda_2}}d+$

$\frac{\eta-\theta s}{\lambda_2-1}\left[(\lambda_1-\lambda_2 t_1)-\lambda_1^{-\frac{1}{\lambda_2}+1}(\lambda_1-\lambda_2 t_1)^{\frac{1}{\lambda_2}}\right]$

(37)

Let $K(t_1,s)$ be the total cost per unit time. Since the total cost is the sum of the set up cost, cost of the units, the inventory holding cost. Therefore the total cost is

$$K(t_1,s)=\frac{A}{T}+\frac{CQ}{T}+\frac{h}{T}\left(\int_0^{t_1} I(t)dt+\int_{t_1}^{T} I(t)dt\right) \tag{38}$$

Substituting the value of I (t), Q and S given in equation's (23), (24), (25) and (26) in equation (27), one can $K(t_1,s)$ as

$$K(t_1,s)=\frac{A}{T}+\frac{C}{T}\ln\left(\frac{\alpha}{\alpha-\beta t_1}\right)^{\frac{1}{\beta}}$$
$$+\frac{h}{T}\left\{-\int_0^{t_1}\left[(\lambda_1-\lambda_2 t)^{\frac{1}{\lambda_2}}\left[\int_t^{t_1}\frac{1}{\alpha-\beta u}(\lambda_1-\lambda_2 t)^{-\frac{1}{\lambda_2}}du\right]\right]dt\right.$$
$$+\frac{s}{\lambda_2+1}\left[\frac{\lambda_1^{\frac{1}{\lambda_2}+1}}{(\lambda_1-\lambda_2 t_1)^{\frac{1}{\lambda_2}}}-\frac{(\lambda_1-\lambda_2 t_2)^{\frac{1}{\lambda_2}}}{(\lambda_1-\lambda_2 t_1)^{\frac{1}{\lambda_2}}}(\lambda_1-\lambda_2 t_2)\right]$$
$$+\frac{\eta-\theta s}{(\lambda_2-1)(\lambda_2+1)}\left[(\lambda_1-\lambda_2 t_1)^{1-\frac{1}{\lambda_2}}(\lambda_1-\lambda_2 t_2)^{1+\frac{1}{\lambda_2}}-\lambda_1^{\frac{1}{\lambda_2}+1}(\lambda_1-\lambda_2 t_1)^{1-\frac{1}{\lambda_2}}\right]$$
$$+\frac{\eta-\theta s}{(\lambda_2-1)}\left[\lambda_1 t_2-\frac{\lambda_2 t_2^2}{2}\right] \tag{39}$$

Let $P(t_1,s)$ be the profit rate function. Since the profit rate function is the total revenue per unit minus total cost per unit time, we have

$P(t_1,s)=s(\eta-\theta s)-K(t_1,s)$

Where, $K(t_1,s)$ is defined in (36)

Optimal pricing and ordering policies of the model

In this section we obtain the optimal policies of the model under study. To find the optimal values of t_1, we equate the first order partial derivatives of $P(t_1,s)$ given equation in (29) with respect to t_1 and s and equate them to zero. The condition for maximization of $P(t_1,s)$ is

$$D=\begin{vmatrix}\frac{\partial^2 P(t_1,s)}{\partial t_1^2} & \frac{\partial^2 P(t_1,s)}{\partial t_1\partial s}\\ \frac{\partial^2 P(t_1,s)}{\partial t_1\partial s} & \frac{\partial^2 P(t_1,s)}{\partial s^2}\end{vmatrix}<0$$

Differentiate $P(t_1,s)$ with respect to $t_1$and equating to zero, one can get

$$C\alpha {t_1}^{\beta-1}-\mathrm{h}\left\{-\frac{\partial}{\partial t_1}\left[\int_0^{t_1}\left[(\lambda_1-\lambda_2 t)^{\frac{1}{\lambda_2}}\left[\int_t^{t_1}\frac{1}{\alpha-\beta u}(\lambda_1-\lambda_2 t)^{-\frac{1}{\lambda_2}}\right]\right]dt\right]\right.$$
$$\left.+\frac{1}{(\lambda_2+1)}\left[\left(\lambda_1^{\frac{1}{\lambda_2}+1}-(\lambda_1-\lambda_2 T)^{\frac{1}{\lambda_2}+1}\right)\frac{1}{\alpha-\beta t_1}(\lambda_1-\lambda_2 t_1)^{-\frac{1}{\lambda_2}}\right]\right\}=0 \tag{40}$$

Differentiate $P(t_1,s)$with respect to 's' and equating to zero, one can get

$$\mathrm{T}\,(2s\theta-\eta)+\mathrm{h}\left\{\frac{\theta}{\lambda_2-1}\left[\frac{\lambda_1^{-\frac{1}{\lambda_2}+1}(\lambda_1-\lambda_2 T)^{\frac{1}{\lambda_2}+1}-{\lambda_1}^2}{\lambda_2+1}+\left[\lambda_1 T-\frac{\lambda_2 T^2}{2}\right]\right]\right\}=0 \tag{41}$$

Solving the equations (30) and (31), we obtain the optimal time at which the replenishment is to be stopped t_1^* of t_1 and the optimal unit selling price s^* of s. The optimum ordering quantity Q^* of Q in the cycle of length T is obtained by substituting the optimal values of t_1 in (25).

$$Q^*=\ln\left(\frac{\alpha}{\alpha-\beta t_1}\right)^{\frac{1}{\beta}} \tag{42}$$

Numerical illustration of the model with shortages

In this section, we discuss a numerical illustration of the model. For demonstrating the solution procedure of the model, let the inventory system without shortages has the following parameter values:

A =500, 550, 600, 650; λ_1 = 100, 125, 150, 175
C = 11, 11.5, 12, 12.5; λ_2 = 5, 5.5, 6, 6.5
h = 5, 5.5, 6, 6.6; η = 15, 16, 17, 18
α = 12, 12.5, 13, 13.5; θ = 0.5, 0.6, 0.7, 0.8
β = 3, 3.25, 3.5, 3.75; n = 5, 5.5, 6, 6.5
T = 12.

The values of above parameters are varied further to observe the trend in optimal policies and the results obtained are shown in Table 3. Substituting these values the optimal ordering quantity Q*, production time, optimal selling price S* and optimal profit per unit time are computed and presented in Table 3.

From Table 3. it is observed that the deterioration rate parameters and production rate parameters have a tremendous influence on the optimal values of the model.

As the ordering cost 'A' increases from 500 to 650, the cost parameter 'C' increases from 11 to 12 and the deteriorating parameter 'λ_1' various from 100 to 175, the optimal ordering quantity Q^*, the optimal production time $t_1^*$are decreasing and total profit K^* are increasing, the optimal selling price s^* decreases.

As the holding cost 'h' increases from 5 to 6.5, the optimal ordering quantity Q^*, the optimal production time t_1^*, and the optimal selling price s^* are decreases. The total profit P* increases. As the production rate parameter 'α' varies from 12 to 13.5, the optimal ordering quantity Q^* and the total profit P* are increasing, the optimal production time t_1^* and optimal selling price s^* are decreasing. Another production rate parameter 'β' various from 3 to 3.75, the optimal ordering quantity Q^* and total profit P* are increasing, the optimal production time t_1^* and the optimal selling price s^* are decreasing.

As the deteriorating parameter 'λ_2' varies from 5 to 6.5, the optimal selling price s^*, the optimal production time t_1^* and total profit P* are increasing, the optimal ordering quantity Q* decreases. As the indexing parameter 'n' increases from 5 to 6.5, the optimal production time t_1^*, the optimal selling price s^* and total profit P^* are decreasing, the optimal ordering quantity Q* increases.

Table 3: Optimal values of production schedule, selling price and production quantity for given values of the parameters and costs

A	C	h	T	α	β	λ_1	λ_2	n	η	θ	t_1	s	Q	P
500	11	5	12	12	3	100	5	5	15	0.5	3.023	64.389	331.362	413.939
550	11	5	12	12	3	100	5	5	15	0.5	3.021	63.448	330.802	415.765
600	11	5	12	12	3	100	5	5	15	0.5	3.019	62.506	330.243	417.591
650	11	5	12	12	3	100	5	5	15	0.5	3.017	61.564	329.683	419.417
500	11.5	5	12	12	3	100	5	5	15	0.5	3.005	62.268	325.561	418.052
500	12	5	12	12	3	100	5	5	15	0.5	2.987	60.263	319.791	421.94
500	12.5	5	12	12	3	100	5	5	15	0.5	2.969	58.369	314.07	425.612
500	11	5.5	12	12	3	100	5	5	15	0.5	3.023	60.722	331.562	413.154
500	11	6	12	12	3	100	5	5	15	0.5	3.024	57.666	331.762	412.369
500	11	6.5	12	12	3	100	5	5	15	0.5	3.024	55.08	331.962	411.585

500	**11**	**5**	**12**	**12.5**	**3**	**100**	**5**	**5**	**15**	**0.5**	2.987	64.004	333.201	414.693
500	**11**	**5**	**12**	**13**	**3**	**100**	**5**	**5**	**15**	**0.5**	2.954	63.605	335.055	415.47
500	**11**	**5**	**12**	**13.5**	**3**	**100**	**5**	**5**	**15**	**0.5**	2.922	63.195	336.925	416.267
500	**11**	**5**	**12**	**12**	**3.25**	**100**	**5**	**5**	**15**	**0.5**	2.798	62.354	340.094	417.888
500	**11**	**5**	**12**	**12**	**3.5**	**100**	**5**	**5**	**15**	**0.5**	2.623	59.858	350.731	422.726
500	**11**	**5**	**12**	**12**	**3.75**	**100**	**5**	**5**	**15**	**0.5**	2.484	56.799	363.724	428.657
500	**11**	**5**	**12**	**12**	**3**	**125**	**5**	**5**	**15**	**0.5**	3.023	63.661	331.399	413.795
500	**11**	**5**	**12**	**12**	**3**	**150**	**5**	**5**	**15**	**0.5**	3.023	63.238	331.421	413.708
500	**11**	**5**	**12**	**12**	**3**	**175**	**5**	**5**	**15**	**0.5**	3.023	62.959	331.436	413.65
500	**11**	**5**	**12**	**12**	**3**	**100**	**5.5**	**5**	**15**	**0.5**	3.023	64.51	331.356	413.962
500	**11**	**5**	**12**	**12**	**3**	**100**	**6**	**5**	**15**	**0.5**	3.023	64.649	331.349	413.989
500	**11**	**5**	**12**	**12**	**3**	**100**	**6.5**	**5**	**15**	**0.5**	3.022	64.814	331.341	414.02
500	**11**	**5**	**12**	**12**	**3**	**100**	**5**	**5.5**	**15**	**0.5**	3.023	63.799	331.401	413.812
500	**11**	**5**	**12**	**12**	**3**	**100**	**5**	**6**	**15**	**0.5**	3.023	63.306	331.434	413.702
500	**11**	**5**	**12**	**12**	**3**	**100**	**5**	**6.5**	**15**	**0.5**	3.023	62.889	331.463	413.608
500	**11**	**5**	**12**	**12**	**3**	**100**	**5**	**5**	**16**	**0.5**	3.024	65.266	331.882	412.24
500	**11**	**5**	**12**	**12**	**3**	**100**	**5**	**5**	**17**	**0.5**	3.026	66.142	332.402	410.54
500	**11**	**5**	**12**	**12**	**3**	**100**	**5**	**5**	**18**	**0.5**	3.027	67.019	332.923	408.841
500	**11**	**5**	**12**	**12**	**3**	**100**	**5**	**5**	**15**	**0.6**	3.019	55.53	330.32	417.337
500	**11**	**5**	**12**	**12**	**3**	**100**	**5**	**5**	**15**	**0.7**	3.016	49.203	329.279	420.736
500	**11**	**5**	**12**	**12**	**3**	**100**	**5**	**5**	**15**	**0.8**	3.013	44.457	328.236	424.133

The demand rate parameter 'η' increases from 15 to 18, the optimal ordering quantity Q^*, the optimal production time t_1^*, and the optimal selling price s^* are increasing and the total profit P* decreases. Another demand rate parameter 'θ' increases from 0.5 to 0.8, the optimal values of t_1^*, s^*, Q^* and are decreasing and P* increasing.

Sensitivity analysis of the model without shortages

The sensitivity analysis is carried to explore the effect of changes in model parameters and costs on the optimal policies, by varying each parameter (-15%, -10%, -5%, 0%, 5%, 10%, 15%) at a time for the model under study. The results are presented in Table 4. The relationship between the parameters and the optimal values of the production schedule is shown in Figure 2.

From Table 4 it is observed that the production time t_1^* is moderately sensitive to C, h, η and θ and slightly sensitive to the changes in other parameter values. The optimal ordering quantity Q^* is highly sensitive to the cost parameters C, h and demand parameters η and θ, moderately sensitive to the indexing parameter 'n' and less sensitive to the other parameters.

For example, increasing C by 15% results in 28.62% decrease in the quantity produced. The optimal selling price s^* are highly sensitive to the demand rate parameters θ. A 15% increase in θ results in 33.11% decrease in s^* and less sensitive to the other parameters. The total profit P* is highly sensitive to the demand parameter η and θ. A 15% decrease in η results in 22.07% decrease in P* and decrease in θ results in 10.21% increase in P*, moderately sensitive to the parameters 'a' and 'n' and less sensitive to the other parameters.

Table 4 (a): Sensitivity Analysis of the Model – Without Shortages

Variation Parameters	Optimal policies	Change in parameters						
		-15%	-10%	-5%	0%	5%	10%	15%
A	t_1^*	3.025	3.024	3.023	3.023	3.022	3.021	3.02
	s^*	65.802	65.331	64.86	64.389	63.918	63.448	62.977
	Q^*	332.2	331.921	331.641	331.362	331.082	330.802	330.523
	P^*	411.2	412.113	413.026	413.939	414.852	415.765	416.678
C	t_1^*	3.079	3.061	3.042	3.023	3.003	2.983	2.964
	s^*	72.289	69.496	66.865	64.389	62.062	59.875	57.822
	Q^*	350.431	344.123	337.753	331.362	324.982	318.642	312.365
	P^*	398.621	404.038	409.139	413.939	418.451	422.691	426.673
h	t_1^*	3.022	3.022	3.022	3.023	3.023	3.023	3.023
	s^*	71.508	68.872	66.513	64.389	62.468	60.722	59.127
	Q^*	331.062	331.162	331.262	331.362	331.462	331.562	331.662
	P^*	415.116	414.724	414.331	413.939	413.546	413.154	412.762
α	t_1^*	3.169	3.117	3.068	3.023	2.98	2.941	2.904
	s^*	65.483	65.218	64.827	64.389	63.925	63.442	62.944
	Q^*	324.723	326.972	329.168	331.362	333.57	335.801	338.055
	P^*	411.534	412.26	413.072	413.939	414.847	415.787	416.753
β	t_1^*	3.62	3.385	3.188	3.023	2.881	2.76	2.655
	s^*	67.196	66.359	65.43	64.389	63.218	61.895	60.399
	Q^*	319.109	322.806	326.861	331.362	336.396	342.055	348.43
	P^*	408.482	410.112	411.918	413.939	416.212	418.778	421.678

Table 4 (b): Sensitivity Analysis of the Model – Without Shortages

Variation Parameters	Optimal policies	Change in parameters						
		-15%	-10%	-5%	0%	5%	10%	15%
λ_1	t_1^*	3.022	3.022	3.023	3.023	3.023	3.023	3.023
	s^*	65.16	64.857	64.604	64.389	64.204	64.042	63.9
	Q^*	331.324	331.339	331.351	331.362	331.371	331.379	331.386
	P^*	414.086	414.029	413.98	413.939	413.903	413.871	413.843
λ_2	t_1^*	3.023	3.023	3.023	3.023	3.023	3.023	3.023
	s^*	64.234	64.283	64.335	64.389	64.448	64.51	64.577
	Q^*	331.369	331.367	331.364	331.362	331.359	331.356	331.353
	P^*	413.909	413.918	413.928	413.939	413.95	413.962	413.975
n	t_1^*	3.022	3.022	3.022	3.023	3.023	3.023	3.023
	s^*	65.535	65.111	64.731	64.389	64.08	63.799	63.542
	Q^*	331.289	331.316	331.34	331.362	331.382	331.401	331.418
	P^*	414.176	414.089	414.011	413.939	413.873	413.812	413.755
[illegible]	t_1^*	3.019	3.02	3.021	3.023	3.024	3.025	3.026
	s^*	62.417	63.075	63.732	64.389	65.047	65.704	66.361
	Q^*	330.19	330.581	330.971	331.362	331.752	332.142	332.533
	P^*	417.762	416.488	415.213	413.939	412.664	411.39	410.115
θ	t_1^*	3.025	3.024	3.023	3.023	3.022	3.021	3.02
	s^*	73.769	70.295	67.187	64.389	61.858	59.557	57.456
	Q^*	332.142	331.882	331.622	331.362	331.101	330.841	330.581
	P^*	411.39	412.24	413.089	413.939	414.788	415.638	416.488

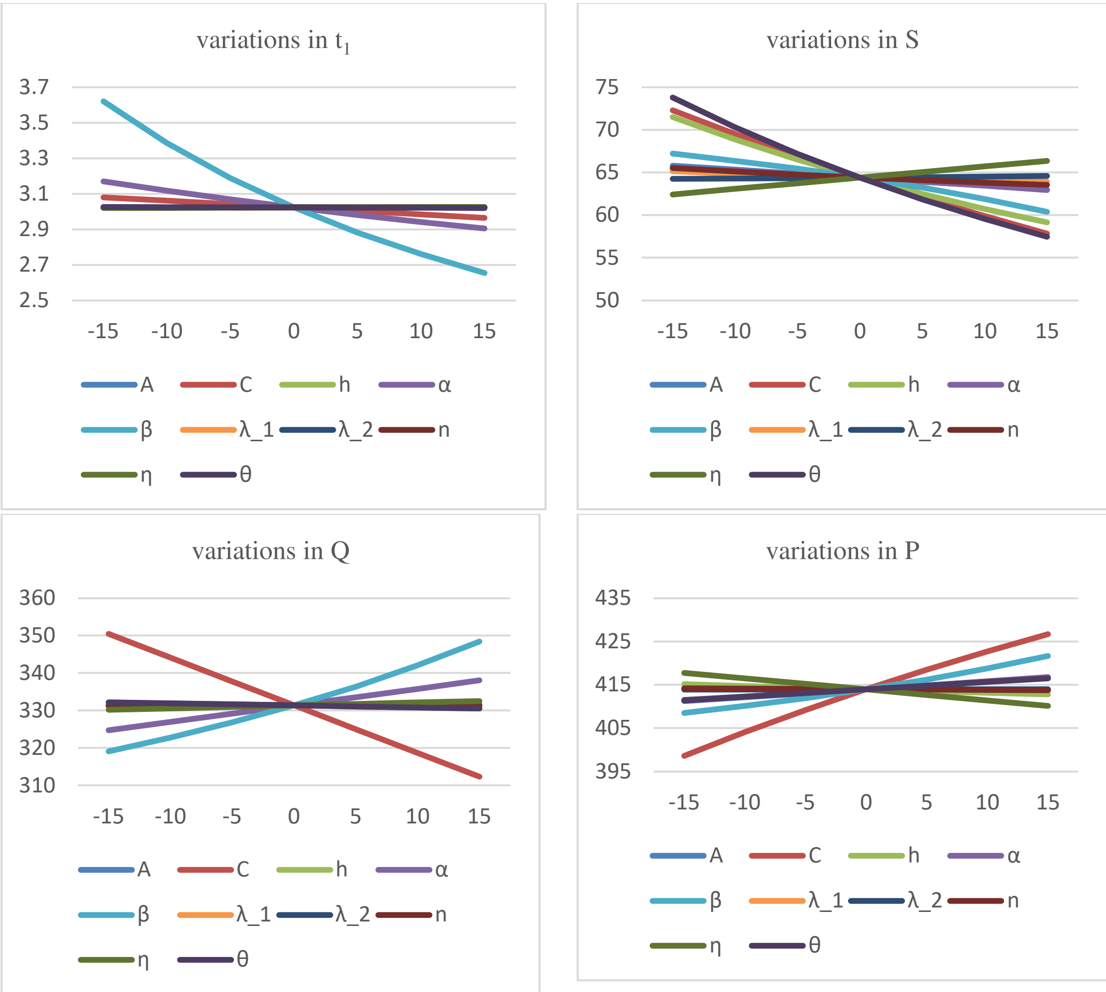

Fig 4: Relationship between parameters and optimal values without shortages

Conclusion

In this paper a novel and trivial application of Generalized Pareto Distribution is considered for developing the production scheduling with suitable cost considerations. Here it is assumed that the production process is random and follows Generalized Pareto Distribution. The Generalized Pareto Rate of Production and deterioration includes constant as well as time dependent rates. The sensitivity analysis of the model revels that the distribution of deterioration and production have tremendous influence on selling price, optimal production quantity, production up time and production down times. It is also observed that allowed reduce the cost of production and utility the resources more efficiently. It is also feasible to develop Economic Production Quantity models with Generalized Pareto Rate of production and deterioration having influence with multiple commodity which will be taken up else ware.

References

1. *Aggarwal, S. P. (1978). 'A note on an order level inventory model for system with constant rate of deterioration', OPSEARCH, Vol. 15, No. 4, pp.184–187.* ***(www.springer.com)***
2. *Cohen, M. A. (1977). 'Joint pricing and ordering for policy exponentially decaying inventories with known demand', Naval Research Logistics. Q, Vol. 24, pp. 257-268.* ***(www.onlinelibrary.wiley.com)***
3. *Covert, R.P. and Philip, G.C. (1973). 'An EOQ model for items with Weibull distribution deterioration', AIIE. TRANS, Vol.5, pp. 323-326.* ***(http://www.aii.ac.in)***
4. *Ghare, P. M and Schrader, G. F. (1963). 'A model for exponentially decaying inventories', Journal of Industrial engineering, Vol. 14, pp. 238-2430.* ***(www.hindawi.com)***
5. *Giri, B. C and Chaudhuri, K. S. (1999). 'An economic production lot-size model with shortages and time dependent demand', IMA Journal of Management Mathematics, Vol. 10, No.3, pp. 203-211.* ***(academic.oup.com)***
6. *Goel, V. P and Aggarwal, S. P. (1980). 'Pricing and ordering policy with general Weibull rate deteriorating inventory', Indian journal Pure and Applied Mathematics, Vol. 11(5), pp. 618 -627.* ***(www.springer.com)***
7. *Goyal, S. K and Giri, B. C. (2003). 'The production- inventory problem of a product with varying demand, production and deterioration rates', European Journal of Operational Research, Vol. 147, No. 3, pp. 549-557.* ***(www.sciencedirect.com)***
8. *Lakshmana Rao, A., Santhi Kumar, R. and Neela Rao, B. (2014). 'Some EPQ model for Weibull deterioration items with selling price dependent demand', International Journal of Soft Computing and Engineering, Vol. 4, Issue 1, pp. 51-56.* ***(www.ijsce.org)***
9. *Nahmias, S. (1974). 'Inventory depletion management when the field life is random', Mgmt. Sci., Vol. 20, pp. 1276-1283.*
10. *Pal, M. (1990). 'An inventory model for deteriorating items when demand is random', Calcutta Statistical Association Bulletin, Vol. 39, pp. 201-207.* ***(www.journals.sagepub.com)***
11. *Pentico, D. W. and Drake, M. J. (2011). 'A survey of deterministic models for the EOQ and EPQ with partial backordering', European Journal of Operational Research, Vol. 214, Issue. 2, pp. 179-198.* ***(www.sciencedirect.com)***
12. *Raafat, F. (1991). 'Survey of literature on continuously deteriorating inventory models', Journal of the Operational Research Society, Vol. 42, No. 1, pp. 27-37.* ***(www.tandfonline.com)***
13. *Ruxian, LI., Lan, H. and Mawhinney, R. J. (2010) 'A review on deteriorating inventory study', Journal of Service Science Management, Vol. 3, No. 1, pp. 117-129. (www.scirp.org)*
14. *Shah, Y. and Jaiswal, M. C. (1977). 'An order level inventory model for a system with a constant rate of deterioration', OPSEARCH, Vol. 14, pp. 174-184.* ***(www.springer.com)***
15. *Sridevi, (2010). 'Inventory model for deteriorating items with Weibull rate of replenishment and selling price dependent demand, International Journal of Operational Research, Vol. 9(3), pp. 329-349.* ***(www.inderscience.com)***
16. *Srinivasa Rao, K., Punyavathi, B, 'On an EPQ model with generalized pareto rate of replenishment and deterioration with constant demand', International Journal of Science & Engineering Research', 2020, Vol.11, Issue1, ISSN 2229-5518.* ***(www.ijesr.org)***
17. *Srinivasa Rao, K., Srinivas. Y., Narayana, B. V. S. and Gopinath, Y. (2009). 'Pricing and ordering policies of an inventory model for deteriorating items having additive exponential lifetime', Indian Journal of Mathematics and Mathematical Sciences, Vol. 5(1), pp. 9-16.* ***(www.serialsjournals.com)***
18. *Srinivasa Rao, K., Uma Maheswara Rao, S. V and Venkata Subbaiah, K. (2011). 'Production inventory models for deteriorating items with production quantity dependent demand and Weibull decay', International Journal of Operational Research, Vol.11, No.1, pp. 31-53.* ***(www.inderscience.com)***

19. *Tadikamalla, P.R. (1978). 'An EOQ inventory model for items with gamma distributed deteriorating', AIIE TRANS, Vol. 10, pp. 100-103.**(www.aii.ac.i)***
20. *Venkata Subbaiah, K., Uma Maheswara Rao, S.V. and Srinivasa Rao, K. (2011). 'An inventory model for perishable items with alternating rate of production', International Journal of Advanced Operations Management, Vol. 3, No. 1, pp. 66-87.**(www.inderscience.com)***
21. *Srinivasa Rao, K, Punyavathi,B (2020)."On an EPQ model with generalized pareto rate of replenishment and deterioration and constant demand", International journal of Scientific &Engineering Research, Vol.11, Issue 1, January.**(www.ijser.org)***
22. *Srinivasa Rao, K, Punyavathi,B (2020). "Inventory model with generalized pareto rate of replenishment having time dependent demand", International Journal of Scientific Research in Mathematical and Statistical Sciences, Vol.7, Issue 1, pp. 92-106, February. **(www.isroset.org)***